مقاومت ۳

مسایل فصل چهارم

کرنش صفحه ای-تنش صفحه ای و توابع تنش ایری

شماره۱:

حالت تنش زیر داده شده است؛

$$\sigma_x = c_1yx^3 - 2c_2xy + c_3y$$

$$\sigma_y = c_1xy^3 - 2c_1x^3y$$

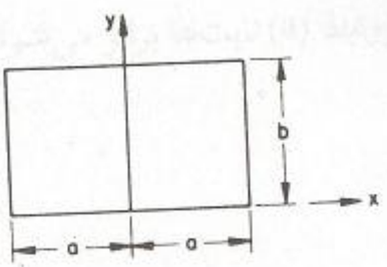
$$\tau_{xy} = -\frac{2}{3}c_1x^2y^2 + c_2y^2 - \frac{1}{2}c_1x^4 + c_4$$

که در آن c_1 تا c_4 مقادیر ثابتی هستند.

(a) نشان دهید که این میدان تنش حلی برای ورق نازکی مطابق شکل است.

(b) تابع تنش مربوط به آن را پیدا کنید.

(c) نیروهای سطحی در لبه‌های y=0 و y=b ورق را بدست آورید.

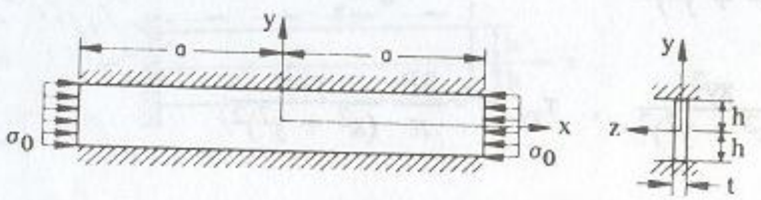


شماره ۲:

مسئله ۱:

یک ورق فولادی نازک و بلند، به ضخامت t، ارتفاع 2h و طول 2a در شکل نشان داده شده است. ورق در دو انتها تحت تنشی یکنواخت $\sigma_\circ$ قرار دارد. اگر دو لبه $y = \pm h$ ورق به دو دیواره صلب ثابت شده باشد، نشان دهید که تغییر مکانها به صورت زیر است. (از روش معکوس استفاده نمائید).

$$u = \frac{1-\nu^2}{E}\sigma_\circ x \quad , \quad v = 0 \quad , w = \frac{\nu\,(1+\nu)}{E}\sigma_\circ z$$



مسئله ۲:

Show that the function

$$F = \frac{q}{20c^3}[10x^2(2y^3 - 3cy^2) - 2y^2(2y^3 - 5cy^2 + 4c^2y - c^3)]$$

may be employed as a stress function. For the plane region $0 \le x \le L,\ 0 \le y \le c$, determine the stress boundary conditions, and describe fully the plane problem for which the stress function serves as the solution for equilibrium.

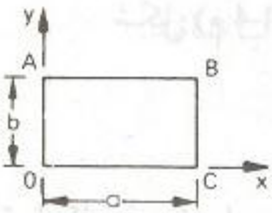
شماره۳:

یک ورق چهارگوش مطابق شکل، به ابعاد a و b و ضخامت t مفروض است.

(a) تنش‌های σ_x . σ_y و τ_{xy} را با فرض تابع تنش به صورت $\phi = c_1x^3y$ (c_1 مقدار ثابتی است) بدست آورید.

(b) توزیع تنش‌ها را روی مرزهای ورق رسم کنید.

(c) نیروهای منتجه تنش روی اضلاع ورق را پیدا کنید.



شماره۴:

نیروی قائم P روی لبه افقی یک سطح نیم بی نهایت به ضخامت واحد مطابق شکل وارد می‌آید. نشان دهید که با فرض تابع تنش به صورت؛

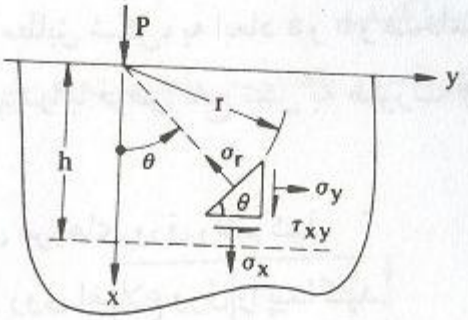
$$\phi = -\frac{P}{\pi} y \, tg^{-1}\left(\frac{y}{x}\right)$$

میدان تنش زیر در ورق حکمفرماست؛

$$\sigma_x = -\frac{2P}{\pi}\frac{x^3}{(x^2+y^2)^2}$$

$$\sigma_y = -\frac{2P}{\pi}\frac{xy^2}{(x^2+y^2)^2} \quad , \quad \tau_{xy} = -\frac{2P}{\pi}\frac{yx^2}{(x^2+y^2)^2}$$

توزیع تنش σ_x و τ_{xy} را در عمق h رسم کنید.

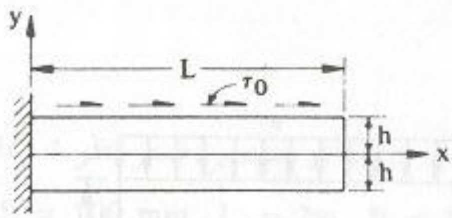


شماره۵:

یک تیر کنسول نازک مطابق شکل مفروض است. در سطح y = +h تیر تنش برشی τ_0 حکمفرماست و حال آنکه دو سطح y = -h بدون بار هستند. نشان دهید که تابع تنش؛

$$\phi = \frac{1}{4}\tau_0 \left(xy - \frac{xy^2}{h} - \frac{xy^2}{h^2} + \frac{Ly^2}{h} + \frac{Ly^3}{h^2}\right)$$

شرایط لازم برای این مسأله را ارضاء می‌کند.



شماره۶:

یک ورق نازک مربع شکل به ضلع a مفروض است. برای تابع تنش؛

$$\phi = \frac{P}{a^2}\left(\frac{1}{2}x^2y^2 - \frac{1}{6}y^4\right)$$

میدان تنش در ورق را پیدا کرده و در امتداد مرزهای ورق رسم کنید.
P در رابطه فوق بار گسترده یکنواخت بر واحد طول است. مبدأ مختصات x و y را در گوشه پائین و سمت چپ ورق انتخاب نمائید.

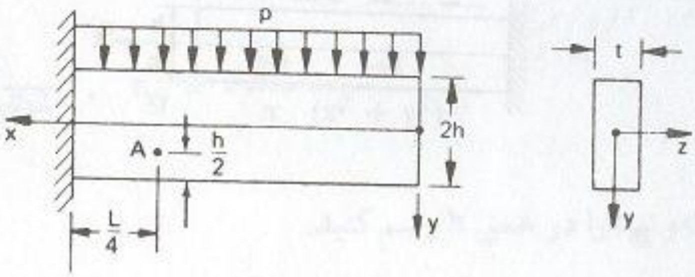
شماره۷:

یک تیر کنسول نازک مطابق شکل تحت بار قرار دارد. میدان تنش در ورق به صورت زیر مفروض است؛

$$\sigma_x = -\frac{M_z y}{I} = -\frac{P}{2I}x^2y \quad , \quad \sigma_y = \sigma_y(x,y) \quad , \quad \tau_{xy} = \tau_{xy}(x,y)$$

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$

مؤلفه‌های تنش σ_y و τ_{xy} را پیدا کنید.



شماره۸:

نشان دهید که برای حالت تنش صفحه‌ای، وقتی نیروهای حجمی وجود نداشته باشند، روابط تعادل برحسب مؤلفه‌های تغییر مکان u و v به صورت زیر است؛

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1+\nu}{1-\nu}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

$$\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} + \frac{1+\nu}{1-\nu}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

شماره۹:

نشان دهید که میدان تنش زیر در یک تیر کنسول تحت بار یکنواخت،

امکان‌پذیر است؛

$$\sigma_x = -\frac{P}{10I}(5x^2 + 2h^2)y + \frac{P}{3I}y^3$$

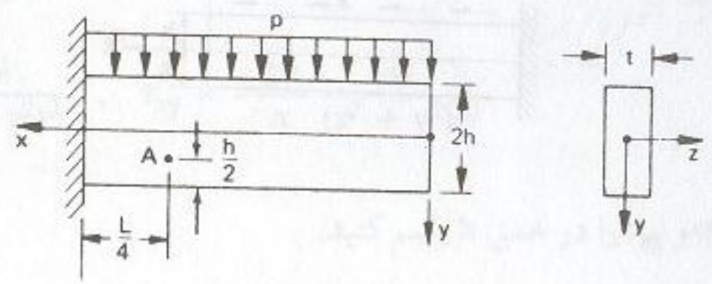
$$\sigma_y = -\frac{P}{6I}(2h^3 - 3h^2y + y^3)$$

$$\tau_{xy} = -\frac{Px}{2I}(h^2 - y^2)$$

که در آن $I = \frac{2th^3}{3}$ است و نیروهای حجمی صفر نظر شده‌اند.

برای حالت $P = 10$ KN/m ، $L = 2$m ، $h = 100$ mm ، $t = 40$ mm ، $\nu = 03$ و

$E = 200$ GPa ، مقدار و جهت ماکزیمم کرنش اصلی در نقطهٔ A را بدست آورید.



شماره۱۰:

Investigate what problem of plane stress is solved by the stress function

$$\phi = \frac{3F}{4c}\left(xy - \frac{xy^3}{3c^2}\right) + \frac{P}{2}y^2$$

شماره۱۱:

Investigate what problem is solved by

$$\phi = -\frac{F}{d^3} xy^2(3d - 2y)$$

شماره۱۲:

applied to the region included in $y = 0$, $y = d$, $x = 0$, on the side x positive.
Show that

$$\phi = \frac{q}{8c^3}\left[x^2\left(y^3 - 3c^2y + 2c^3\right) - \frac{1}{5}y^3\left(y^2 - 2c^2\right)\right]$$

is a stress function, and find what problem it solves when applied to the region included in $y = \pm c$, $x = 0$, on the side x positive.

شماره۱۳:

For a state of plane strain, $\sigma_x = f(y)$. Neglecting body forces, derive the most general equations for σ_x, σ_y, σ_z, and τ_{xy}.

شماره۱۴:

For a state of plane strain in an isotropic body, $\sigma_x = ay^2$, $\sigma_y = -ax^2$, $\tau_{xy} = 0$. The body forces and temperature are zero. Using small-displacement elasticity theory, compute the displacement components $u(x, y)$ and $v(x, y)$ (a is a constant).

شماره۱۵:

For a state of plane strain in an isotropic body,

$$\sigma_x = ay^2 + bx, \qquad \sigma_y = -ax^2 + by, \qquad \tau_{xy} = -b(x + y)$$

The body forces and temperature are zero. Using small-displacement elasticity theory, compute the displacement components $u(x, y)$ and $v(x, y)$ (a and b constants). (See Section

شماره۱۶:

The following stress array is proposed as a solution to a certain *equilibrium* problem of a plane body bounded in the region $-L/2 \le x \le L/2$, $-h/2 \le y \le h/2$:

$$\sigma_x = Ay + Bx^2y + Cy^3, \qquad \sigma_y = Dy^3 + Ey + F,$$
$$\tau_{xy} = (G + Hy^2)x, \qquad \sigma_z = \tau_{xz} = \tau_{yz} = 0$$

where (x, y, z) are rectangular Cartesian coordinates and $A, B, \ldots, H$ are nonzero constants. Determine the conditions under which this array is a possible equilibrium solution.

It is proposed that the region be loaded such that $\tau_{xy} = 0$ for $y = \pm h/2$, $\sigma_y = 0$ for $y = h/2$, $\sigma_y = -\sigma$ (σ = constant) for $y = -h/2$, and $\sigma_x = 0$ for $x = \pm L/2$. Determine whether the proposed stress array may satisfy these conditions.

شماره۱۷:

A flat plate is in a state of biaxial tension. The principal stresses are σ_x and σ_y (see Fig. P5-2.5). Two electrical strain gages are located as shown. The angle α is given by

$$\cos\alpha = \sqrt{\frac{1}{1+\nu}} \qquad \sin\alpha = \sqrt{\frac{\nu}{1+\nu}}$$

Assume that the material is linearly elastic and isotropic. Prove that the principal stresses may be read directly (except for a constant factor) as the strains in the direction of the two strain gages 1 and 2.

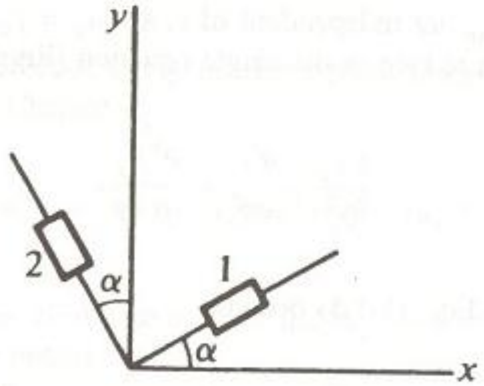
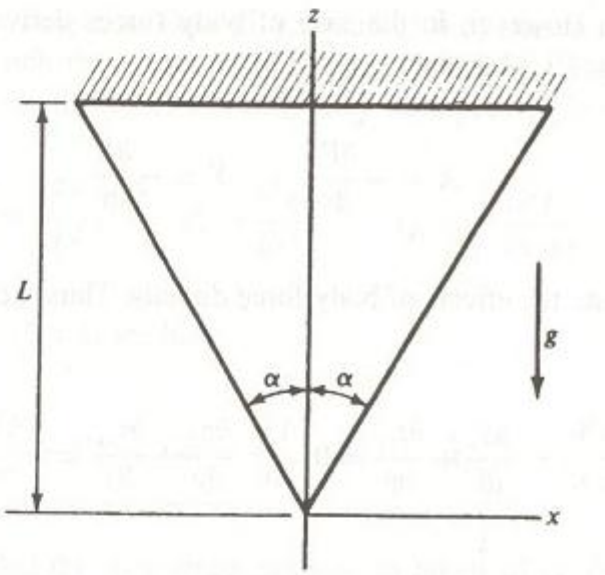


Figure P5-2.5

شماره۱۸:

Consider a wedge hanging vertically in a gravity field of acceleration g (Fig. P5-3.1). The following elasticity solution for the stress problem of the wedge is proposed: $\sigma_x = \sigma_y = \tau_{xy} = \tau_{yz} = 0$, $\sigma_z = \frac{1}{2}\rho g z$, $\tau_{xz} = \frac{1}{2}\rho g x$. Discuss this proposed solution.



شماره۱۹:

Consider a beam in the region $-h/2 \leq y \leq h/2$, $-b/2 \leq z \leq b/2$, $0 \leq x \leq L$. Assume plane stress in the (x, y) plane, with zero body forces. The stress component normal to the plane perpendicular to the x axis is $\sigma_x = -My/I$, where $M = M(x)$ is a function of x only,

and $I = bh^3/12$. Derive expressions for σ_y and τ_{xy} subject to the boundary conditions $\tau_{xy} = 0$ for $y = \pm h/2$ and $\sigma_y = 0$ for $y = h/2$. What restriction, if any, must be placed on M in order that the derived state of stress be compatible? What can be said about σ_y at $y = -h/2$?

شماره۲۰:

Given the following stress state:

$$\sigma_x = C[y^2 + \nu(x^2 - y^2)], \qquad \tau_{xy} = -2C\nu xy$$
$$\sigma_y = C[x^2 + \nu(y^2 - x^2)], \qquad \tau_{yz} = \tau_{xz} = 0$$
$$\sigma_z = C\nu(x^2 + y^2)$$

Discuss the possible reasons for which this stress state may not be a solution of a problem in elasticity.

شماره۲۱:

An infinite plane strip is bounded by the lines $y = \pm 1$. The stresses on the lines $y = \pm 1$ are $\sigma_y = \cos x$, $\tau_{xy} = 0$. There is no body force. By assuming an Airy stress function of the form $f(y)\cos x$, determine σ_x, σ_y, τ_{xy} as functions of (x, y).

شماره۲۲:

The following stress–strain relations pertain to the anisotropic flat thin plate subjected to a state of generalized plane stress:

$$\epsilon_x = S_{11}\sigma_x + S_{12}\sigma_y$$
$$\epsilon_y = S_{12}\sigma_x + S_{22}\sigma_y$$
$$\gamma_{xy} = S_{33}\tau_{xy} \quad (x, y) = \text{rectangular Cartesian coordinates}$$

where S_{11}, S_{22}, S_{33}, S_{12} are elastic constants and where $(\sigma_x, \sigma_y, \tau_{xy})$ and $(\epsilon_x, \epsilon_y, \gamma_{xy})$ are average values of stress and strain through the thickness. Let $(\sigma_x, \sigma_y, \tau_{xy})$ be defined in terms of an Airy stress function F. Show that the defining equation for the Airy stress function F is of the form

$$\left(\frac{\partial^2}{\partial x^2} + \alpha_1 \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 F}{\partial x^2} + \alpha_2 \frac{\partial^2 F}{\partial y^2}\right) = 0 \qquad \text{(a)}$$

where α_1, α_2 are constants. For the case $S_{11} = S_{22} = 1/E$, $S_{12} = -\nu/E$, $S_{33} = 1/G$, show that Eq. (a) reduces to the biharmonic equation.

شماره۲۳:

Let

$$F = ax^2 + by^3 + \sum_{n=1}^{\infty} A_n(y)\cos\left(\frac{n\pi x}{L}\right)$$

be an Airy stress function for a plane, isotropic problem, where a, b, L are constants, and $A_n(y)$ are functions of y, Derive the defining differential equation for the coefficients A_n.

Consider a plane rectangular region $-L \leq x \leq L$, $-C \leq y \leq C$. Assume that no net force or no net couple acts on the sections $x = \pm L$. Discuss how the arbitrary constants in the solution of the differential equation for $A_n(y)$ may be evaluated.

شماره۲۴:

Consider a case of plane stress without body forces in the region $-c \leq y \leq c$, $0 \leq x \leq \ell$ (see Fig. P5-4.9). If the resultant of the stresses in the x direction is zero, the elementary beam formula yields $\sigma_x = My/I$; that is, σ_x is a linear function of y.

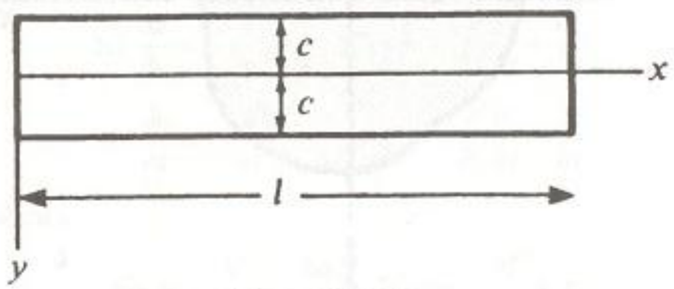


Figure P5-4.9

شماره۲۵:

9. Consider a case of plane stress without body forces in the region $-c \leq y \leq c$, $0 \leq x \leq \ell$ (see Fig. P5-4.9). If the resultant of the stresses in the x direction is zero, the elementary beam formula yields $\sigma_x = My/I$; that is, σ_x is a linear function of y.

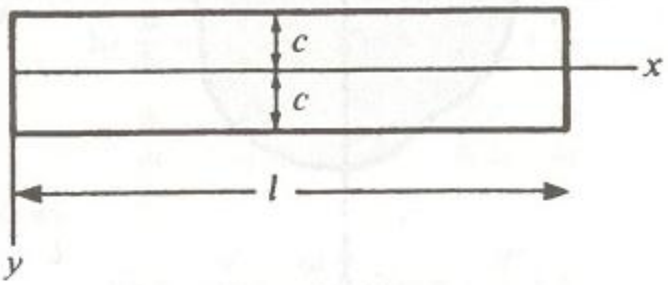


Figure P5-4.9

(a) Let $\sigma_x = F_{yy}$, $\sigma_y = F_{xx}$, $\tau_{xy} = -F_{xy}$. Write the most general expression for $F(x, y)$ that satisfies the equations of equilibrium and yields σ_x as linear function of y in the form $\sigma_x = yf(x)$.

(b) Assuming that the material is isotropic and linearly elastic, write the equation of compatibility for $F(x, y)$ as determined in part (a).

شماره۲۶:

For a plane problem, the stress components in the (x, y) rectangular region $0 \leq x \leq L$, $-c \leq y \leq c$, where L and c are constants, are given by the relations (q = constant)

$$\sigma_x = \frac{qx^3y}{4c^3} + \frac{q}{4c^3}\left(-2xy^3 + \frac{6}{5}c^2xy\right)$$

$$\sigma_y = -\frac{qx}{2} + qx\left(\frac{y^3}{4c^3} - \frac{3y}{4c}\right)$$

$$\tau_{xy} = \frac{3qx^2}{8c^3}(c^2 - y^2) - \frac{q}{8c^3}(c^4 - y^4) + \frac{q}{4c^3} \cdot \frac{3c^2}{5}(c^2 - y^2)$$

(a) Show that these stress components satisfy the equations of equilibrium in the absence of body forces.

(b) Derive the Airy stress function from which these stress components are derivable.

شماره۲۷:

The stress function for a cantilever beam loaded by a shear force P at the free end is

$$F = C_1 xy^3 + C_2 xy$$

(a) Evaluate the constants C_1 and C_2.

(b) Derive the expressions for the displacements u and v.

شماره۲۸:

Apply the stress function $F = -(P/d^3)xy^2(3d - 2y)$ to the region $0 \le y \le d$, $0 \le x$. Determine what kind of problem is solved by this stress function.

شماره۲۹:

Consider the Airy stress function $F = Ax^3y$, where A is a constant and (x, y) are rectangular Cartesian coordinates. Determine the plane elasticity problem that is solved by this function for the region $-a \le x \le a$, $-b \le y \le b$.