بسمه تعالی

تمرینات الاستیسیته

فصل چهارم

مسایل تنش و کرنش صفحه ای و توابع تنش ایری

شماره ۱:

a)

Explicitly show that the fourth-order polynomial Airy stress function

$$A_{40}x^4 + A_{22}x^2y^2 + A_{04}y^4$$

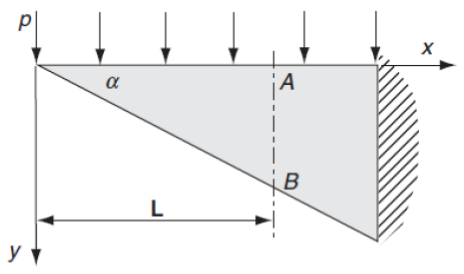
will not satisfy the biharmonic equation unless $3A_{40} + A_{22} + 3A_{04} = 0$.

b)

A triangular plate of narrow rectangular cross-section and uniform thickness is loaded uniformly along its top edge as shown in the following figure. Verify that the Airy stress function

$$\phi = \frac{p\cot\alpha}{2(1-\alpha\cot\alpha)}\left[-x^2\tan\alpha + xy + (x^2+y^2)(\alpha - \tan^{-1}\frac{y}{x})\right]$$

solves this plane problem. For the particular case of $\alpha = 30°$, explicitly calculate the normal and shear stress distribution over a typical cross-section AB and make comparison plots (MATLAB recommended) of your results with those from elementary strength of materials.



$$\sigma_x = 2K\left[\alpha - \tan^{-1}\frac{y}{x} - \frac{xy}{x^2+y^2}\right],\ \sigma_y = 2K\left[\alpha - \tan\alpha - \tan^{-1}\frac{y}{x} + \frac{xy}{x^2+y^2}\right]$$

$$\tau_{xy} = -2K\frac{y^2}{x^2+y^2},\ K = \frac{p\cot\alpha}{2(1-\alpha\cot\alpha)}$$

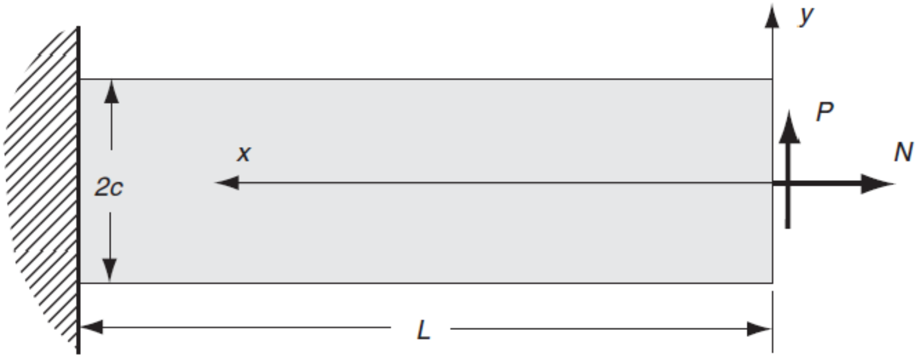
شماره ۲:

Show that the Airy function

$$\phi = \frac{3P}{4c}\left(xy - \frac{xy^3}{3c^2}\right) + \frac{N}{4c}y^2$$

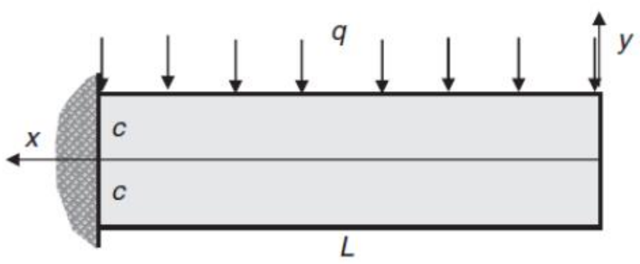
solves the following cantilever beam problem, as shown in the following figure. As usual for such problems, boundary conditions at the ends ($x = 0$ and L) should be formulated only in terms of the *resultant force system*, while at $y = \pm c$ the exact *pointwise* specification should be used. For the case with $N = 0$, compare the elasticity stress field with the corresponding results from strength of materials theory. *Answer*:

$$\sigma_x = -\frac{3Pxy}{2c^3} + \frac{N}{2c}, \ \sigma_y = 0, \ \tau_{xy} = -\frac{3P}{4c}\left(1 - \frac{y^2}{c^2}\right)$$



شماره۳:

The solution to the illustrated two-dimensional cantilever beam problem is proposed using the Airy stress function $\phi = C_1x^2 + C_2x^2y + C_3y^3 + C_4y^5 + C_5x^2y^3$, where C_i are constants. First determine requirements on the constants so that ϕ satisfies the governing equation. Next find the values of the remaining constants by applying exact pointwise boundary conditions on the top and bottom of the beam and integrated resultant boundary conditions on the ends $x = 0$ and $x = L$.

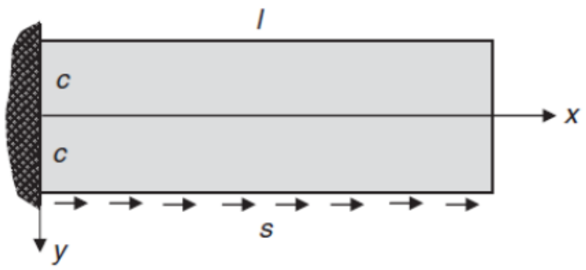


شماره۴:

Verify that the Airy stress function

$$\phi = \frac{s}{4}\left(xy + \frac{ly^2}{c} + \frac{ly^3}{c^2} - \frac{xy^2}{c} - \frac{xy^3}{c^2}\right)$$

solves the problem of a cantilever beam loaded by uniform shear along its bottom edge as shown. Use pointwise boundary conditions on $y = \pm c$ and only resultant effects at ends $x = 0$ and l. Note however, you should be able to show that σ_x vanishes at $x = l$.



شماره۵:

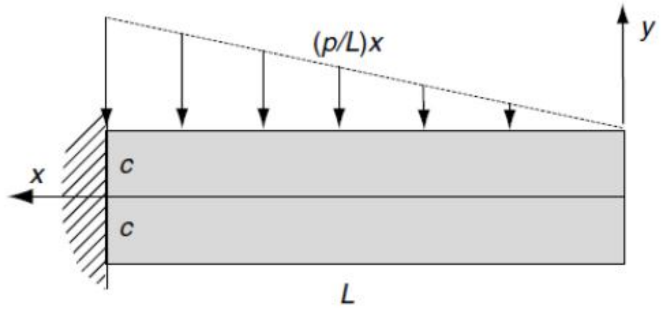
The following stress function

$$\phi = C_1 xy + C_2 \frac{x^3}{6} + C_3 \frac{x^3 y}{6} + C_4 \frac{xy^3}{6} + C_5 \frac{x^3 y^3}{9} + C_6 \frac{xy^5}{20}$$

is proposed to solve the problem of a cantilever beam carrying at uniformly varying loading as shown in the following figure. Explicitly verify that this stress function will satisfy all conditions on the problem and determine each of the constants C_i and resulting stress field. Use resultant force boundary conditions at the beam-ends. *Answers*:

$$C_1 = -\frac{pc}{40L},\ C_2 = -\frac{p}{2L},\ C_3 = -\frac{3p}{4Lc},\ C_4 = \frac{3p}{10Lc},\ C_5 = \frac{3p}{8Lc^3}$$

$$C_6 = -\frac{p}{2Lc^3},\ \sigma_x = \frac{pxy}{20Lc^3}(5x^2 - 10y^2 + 6c^2)$$

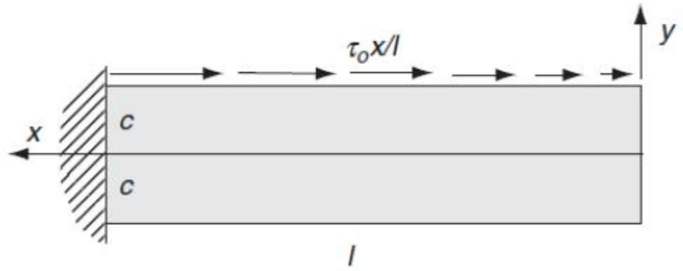


شماره۶:

The cantilever beam shown in the figure is subjected to a distributed shear stress $\tau_o x/l$ on the upper face. The following Airy stress function is proposed for this problem

$$\phi = c_1 y^2 + c_2 y^3 + c_3 y^4 + c_4 y^5 + c_5 x^2 + c_6 x^2 y + c_7 x^2 y^2 + c_8 x^2 y^3$$

Determine the constants c_i and find the stress distribution in the beam. Use resultant force boundary conditions at the ends. (*Answer*: $c_1 = \tau_o c/12l$, $c_2 = \tau_o/20l$, $c_3 = -\tau_o/24cl$, ...)



شماره۷:

By the method of Neou, the Airy stress function

$$F = \frac{p}{60a}\left(5\frac{L^2}{a^2} - 3\right)y^3 + \frac{p}{40a^3}y^5 - \frac{pa}{40L}xy + \frac{p}{20aL}xy^3$$
$$- \frac{p}{40a^3L}xy^5 - \frac{p}{4}x^2 + \frac{3p}{8a}x^2y - \frac{p}{8a^3}x^2y^3 + \frac{p}{12L}x^3$$
$$- \frac{p}{8aL}x^3y + \frac{p}{24a^3L}x^3y^3$$

is obtained for a rectangular beam supported by end shear load and subjected to a triangular load as shown in Fig. P5-7.7. Discuss the validity of the solution.

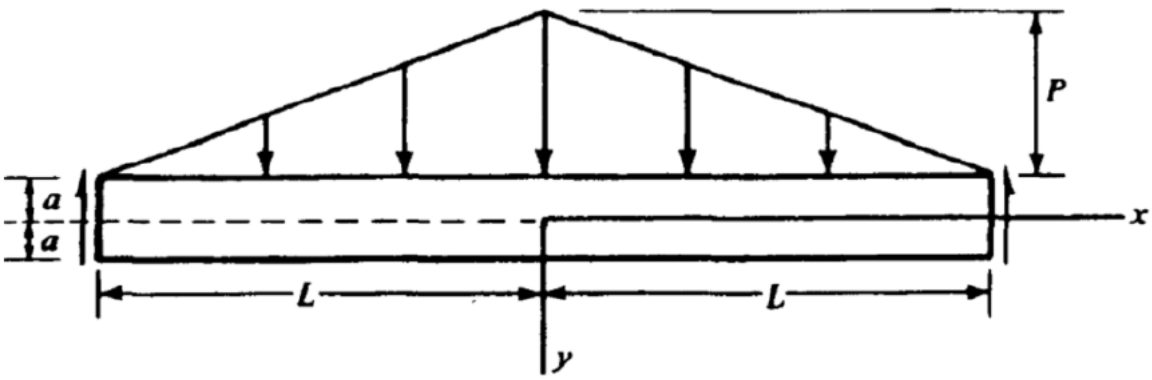


Figure P5-7.7

شماره۸:

a)

The stress function

$$\phi = s\left(\frac{1}{4}xy - \frac{xy^2}{4c} - \frac{xy^3}{4c^2} + \frac{ly^2}{4c} + \frac{ly^3}{4c^2}\right)$$

is proposed as giving the solution for a cantilever ($y = \pm c$, $0 < x < l$) loaded by uniform shear along the lower edge, the upper edge and the end $x = l$ being free from load. In what respects is this solution imperfect? Compare the expressions for the stresses with those obtainable from elementary tension and bending formulas.

b)

The thin homogeneous plane strip of width $2h$ extends a great distance in the $\pm x$ direction Fig. P5-6.9. The plate is rigidly restrained by the fixed walls at $y = \pm h$. The plate is loaded by gravity in the $-y$ direction. The density of the plate is ρ. Assume $\partial/\partial x = u = \tau_{xy} = X = 0$. The plate is made of a material whose stress–strain relations are

$$\epsilon_x = A\sigma_x^3 - B\sigma_y^3, \qquad \epsilon_y = A\sigma_y^3 - B\sigma_x^3, \qquad \gamma_{xy} = C\tau_{xy}$$

where A, B, and C are known constants. Determine formulas for σ_x, σ_y, and v as functions of y and the known constants A, B, C, ρ, g, and h.

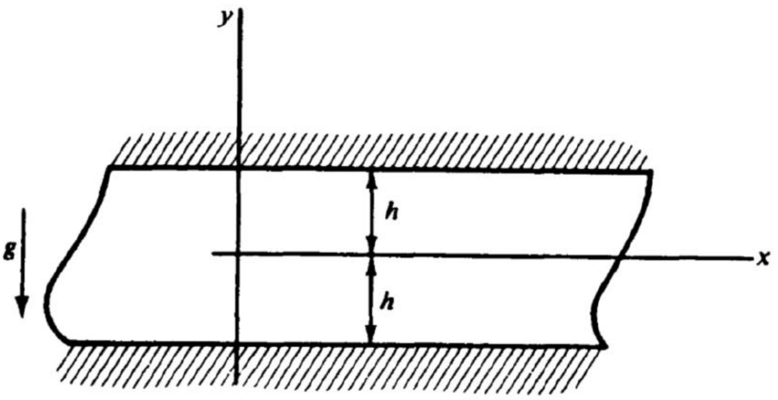


Figure P5-6.9