فصل سیزدهم
پیچش

شماره ۱:

شتمه ۱۲- یک محور مدور به شعاع a گشتناور پیچشی T را متحرک میکند. توزیع تنش برشی را در محور باید نمایی با استفاده از تابع تنش (') مستقیماً با استفاده از تابع تنش m = m(x', x', a') مثلاً را حاصل کنید. m یک ثابت است.

شماره ۲:

6.12. Two bars, one with a square cross section and one with a circular cross section, have equal cross-sectional areas. The bars are subjected to equal twisting moments. Determine the ratio of the maximum shear stresses in the two bars, assuming that they remain elastic.

Ans. $\tau_{\text{max(square bar)}} = 1.36 \tau_{\text{max(circular bar)}}$

شتمه ۱۳- یک محور فولادی با سطح مقطع بیضوی، با شکل است. مقدار و موقعیت تنش برشی جداساز در مقطع و زاویه پیچش بر واحد طول را محاسبه کنید (دارای: $E = 200\text{ GPa}$، $\nu = 0.3$ و $\theta = 3\sqrt{2}/\pi$ میل متر، $a = 4\text{ mm}$) در مورد $\sigma_{\text{max}} = 92/1 \text{ MPa}$

جواب:
6.14. Consider a hollow elliptic cylinder with its outer elliptic surface defined by 
\[(x/h)^2 + (y/b)^2 = 1\] and inner elliptic surface defined by \[(x/(kh))^2 + (y/(kb))^2 = 1\]. Show that
\[
\theta = \frac{(h^2 + b^2)T}{\pi h^3 b^3 (1 - k^4)G}, \quad \tau_{\text{max}} = \frac{2T}{\pi bh^2 (1 - k^4)}
\]
and
\[
\sigma_{xx} = -\frac{2T}{\pi hb^3 (1 - k^4)} y, \quad \sigma_{yy} = \frac{2T}{\pi bh^3 (1 - k^4)} x
\]

Hint: By the theory of hollow torsional members (Boresi and Chong, 1987), the twisting moment \(T\) is related to \(\phi\) by the relation
\[
T = \iint_R 2\phi dA + 2K_1 A_1, \text{ where } \phi = A \left(\frac{x^2}{h^2} + \frac{y^2}{b^2} - 1\right),
\]

\(K_1\) is the value of \(\phi\) on the inner elliptic surface, \(A_1\) is the area bounded by the inner ellipse and \(R\) is the solid region bounded by the inner and outer ellipses.
6.15. Find the maximum shear stress and unit angle of twist of the bar having the cross section shown in Fig. P6.15 when subjected to a torque at its ends of 600 N·m. The bar is made of a steel for which \( G = 77.5 \) GPa.

![Figure P6.15](image)

Referring to Figure 9-2, if we choose a different reference origin that is located at point \((a,b)\) with respect to the given axes, the displacement field would now be given by

\[
\begin{align*}
    u &= -xz(y - b), \\
    v &= xz(x - a), \\
    w &= w(x, y)
\end{align*}
\]

where \(x\) and \(y\) now represent the new coordinates. Show that this new representation leads to an identical torsion formulation as originally developed.

![Figure 9-2](image)
6.16. An aluminum alloy extruded section (Fig. P6.16) is subjected to a torsional load. Determine the maximum torque that can be applied to the member if the maximum shear stress is 75.0 MPa. Neglect stress concentrations at changes in section.

Ans. \( T = 665.4 \text{ N}\cdot\text{m} \)

A circular shaft with a keyway can be approximated by the section shown in the figure. The keyway is represented by the boundary equation \( r = b \), while the shaft has the boundary relation \( r = 2a \cos \theta \). Using the technique of Section 9.4, a trial stress function is suggested of the form

\[
\phi = K(b^2 - r^2)(1 - \frac{2a \cos \theta}{r})
\]

where \( K \) is a constant to be determined. Show that this form will solve the problem and determine the constant \( K \). Compute the two shear stress components \( \tau_x \) and \( \tau_y \).
6.22. An aluminum \((G = 26.7 \text{ GPa})\) torsion bar has the cross section shown in Fig. P6.22. The bar is subjected to a twisting moment \(T = 1356 \text{ m-N.}\)

![Diagram of a torsion bar with dimensions 100 mm, 12 mm, 10 mm, 200 mm, and 12 mm]

Figure P6.22

(a) Determine the maximum shear stress \(\tau_{\text{max}}\) and angle of twist per unit length.
(b) At what location in the cross section does \(\tau_{\text{max}}\) occur? Ignore stress concentrations.

6.4. A square shaft has 42.0-mm sides and the same cross-sectional area as shafts having circular and equilateral triangular cross sections. If each shaft is subjected to a torque of 1.00 kN-m, determine the maximum shear stress for each of the three shafts.

**Ans.** \(\tau_{\text{square}} = 64.89 \text{ MPa}, \quad \tau_{\text{circle}} = 47.82 \text{ MPa}, \quad \tau_{\text{triangle}} = 76.86 \text{ MPa}\)
6.23. Compare the shear stress and the unit angle of twist for three thin-wall sections: a circular tube, a square tube, and an equilateral triangle. The three sections have equal wall thicknesses and equal perimeters.

*Ans.* \[ \tau_{\text{square}} = 1.27 \tau_{\text{circle}}; \quad \tau_{\text{triangle}} = 1.65 \tau_{\text{circle}}; \quad \theta_{\text{square}} = 1.62 \theta_{\text{circle}}; \]

\[ \theta_{\text{triangle}} = 2.74 \theta_{\text{circle}} \]

6.7. A torsion member has an elliptical cross section with major and minor dimensions of 50.0 mm and 30.0 mm, respectively. The yield stress of the material in the torsion member is \( Y = 400 \) MPa. Determine the maximum torque that can be applied to the torsion member based on a factor of safety of \( SF = 1.85 \) using the maximum shear stress criterion of failure.

6.32. The I-beam in Fig. 6.32 is an aluminum alloy (\( E = 72.0 \) GPa and \( G = 27.1 \) GPa) extruded section. It is fixed at the wall and attached rigidly to the thick massive plate at the other end. Determine the magnitude of \( P \) for \( \sigma_{\text{max}} = 160 \) MPa.

*Ans.* \( P = 1.095 \) kN
6.10. A rectangular bar has a cross section such that \( b/h = k \), and it is subjected to a twisting moment \( T \). A cylindrical bar of diameter \( d \) is also subjected to \( T \). Show that the maximum shear stresses in the two bars are equal, provided \( d = 3.441h(kk_2)^{1/3} \) and the bars remain elastic.

2. Show that \( \phi = A(r^2 - a^2) \) solves the torsion problem for the solid or hollow circular shaft. Determine \( A \) in terms of \( G\theta \). Evaluate the maximum shearing stress and the torsional rigidity in terms of \( M_t \) for the solid shaft, and verify that the results are in agreement with those given in any