الاستیسیته

فصل سیزدهم

پیچش

شماره۱:

مسئله ۱۳-۱ یک محور مدور به شعاع a گشتاور پیچشی T را منتقل می‌کند. توزیع تنش برشی را در محور بیابید.
(راهنمایی: با استفاده از تابع تنش $\psi = m(x_1^2 + x_2^2 - a^2)$ مسئله را حل کنید. m یک ثابت است.

6.12. Two bars, one with a square cross section and one with a circular cross section, have equal cross-sectional areas. The bars are subjected to equal twisting moments. Determine the ratio of the maximum shear stresses in the two bars, assuming that they remain elastic.

Ans. $\tau_{\max(\text{square bar})} = 1.36\, \tau_{\max(\text{circular bar})}$

شماره۲:

مسئله ۱۳-۲ یک محور فولادی با سطح مقطع بیضوی، با $a = ۲۴\text{mm}$ و $b = ۱۲\text{mm}$ تحت گشتاور پیچشی ۵۰۰N.m واقع شده است. مقدار و موقعیت تنش برشی حداکثر در مقطع و زاویهٔ پیچش بر واحد طول را محاسبه کنید (داریم: $E = ۲۰۰\text{GPa}$ و $\nu = ۰/۳$).

جواب: $(\sigma_s)_{\max} = ۹۲/۱\ \text{MPa}$ در $x_1 = ۰$ ، $x_2 = \pm ۱۲\text{mm}$ ، $\theta' = ۳/۵۷^\circ/\text{m}$

6.14. Consider a hollow elliptic cylinder with its outer elliptic surface defined by $(x/h)^2 + (y/b)^2 = 1$ and inner elliptic surface defined by $[x/(kh)]^2 + [y/(kb)]^2 = 1$. Show that

$$\theta = \frac{(h^2 + b^2)T}{\pi h^3 b^3 (1 - k^4) G} \qquad \tau_{\max} = \frac{2T}{\pi b h^2 (1 - k^4)}$$

and

$$\sigma_{zx} = -\frac{2T}{\pi h b^3 (1 - k^4)} y, \qquad \sigma_{zy} = \frac{2T}{\pi b h^3 (1 - k^4)} x$$

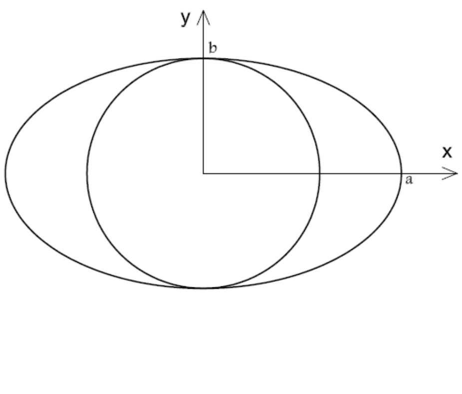
Hint: By the theory of hollow torsional members (Boresi and Chong, 1987), the twisting moment T is related to ϕ by the relation

$$T = \iint_R 2\phi \, dA + 2K_1 A_1, \text{ where } \phi = A\left(\frac{x^2}{h^2} + \frac{y^2}{b^2} - 1\right),$$

K_1 is the value of ϕ on the inner elliptic surface, A_1 is the area bounded by the inner ellipse and R is the solid region bounded by the inner and outer ellipses.

شماره۳:

3. Show that for the same twist, the elliptic section has a greater shearing stress than the inscribed circular section (radius equal to the minor axis b of the ellipse). Which takes the greater torque for the same allowable stress?



6.15. Find the maximum shear stress and unit angle of twist of the bar having the cross section shown in Fig. P6.15 when subjected to a torque at its ends of 600 N·m. The bar is made of a steel for which $G = 77.5$ GPa.

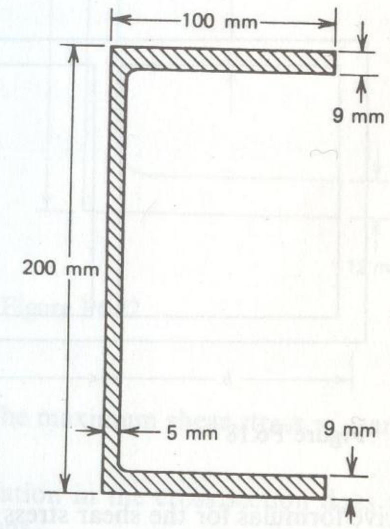


Figure P6.15

شماره۴:

Referring to Figure 9-2, if we choose a different reference origin that is located at point (a,b) with respect to the given axes, the displacement field would now be given by

$$u = -\alpha z(y - b), \quad v = \alpha z(x - a), \quad w = w(x, y)$$

where x and y now represent the new coordinates. Show that this new representation leads to an identical torsion formulation as originally developed.

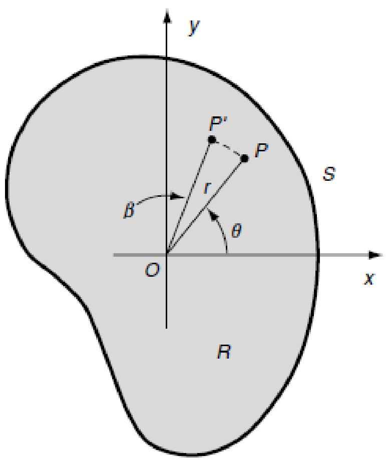


FIGURE 9-2 *In-plane displacements for the torsion problem.*

6.16. An aluminum alloy extruded section (Fig. P6.16) is subjected to a torsional load. Determine the maximum torque that can be applied to the member if the maximum shear stress is 75.0 MPa. Neglect stress concentrations at changes in section.

Ans. $T = 665.4$ N·m

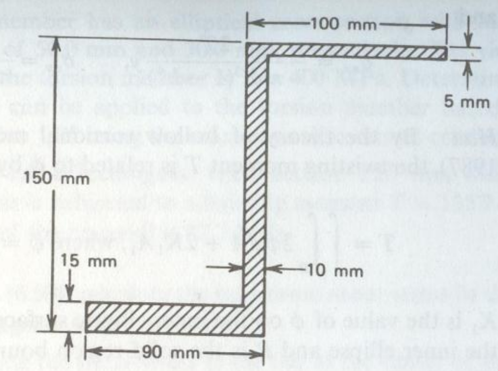


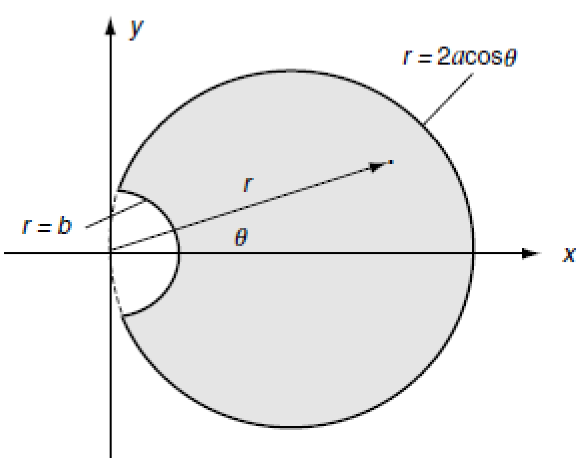
Figure P6.16

شماره۵:

A circular shaft with a keyway can be approximated by the section shown in the figure. The keyway is represented by the boundary equation $r = b$, while the shaft has the boundary relation $r = 2a\cos\theta$. Using the technique of Section 9.4, a trial stress function is suggested of the form

$$\phi = K(b^2 - r^2)\left(1 - \frac{2a\cos\theta}{r}\right)$$

where K is a constant to be determined. Show that this form will solve the problem and determine the constant K. Compute the two shear stress components τ_{xz} and τ_{yz}.



6.22. An aluminum ($G = 26.7$ GPa) torsion bar has the cross section shown in Fig. P6.22. The bar is subjected to a twisting moment $T = 1356$ m·N.

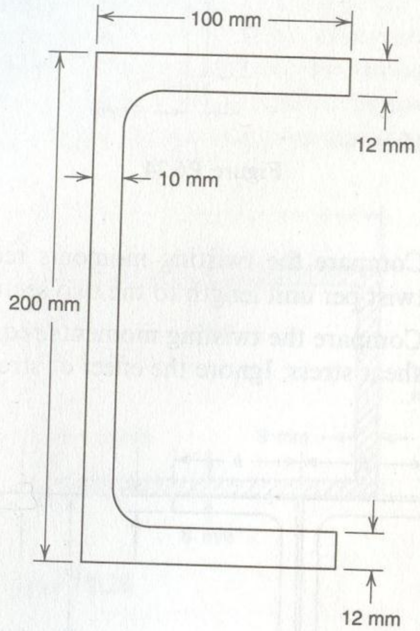


Figure P6.22

(a) Determine the maximum shear stress τ_{max} and angle of twist per unit length.

(b) At what location in the cross section does τ_{max} occur? Ignore stress concentrations.

شماره ۶:

6.4. A square shaft has 42.0-mm sides and the same cross-sectional area as shafts having circular and equilateral triangular cross sections. If each shaft is subjected to a torque of 1.00 kN·m, determine the maximum shear stress for each of the three shafts.

Ans. $\tau_{square} = 64.89$ MPa, $\tau_{circle} = 47.82$ MPa, $\tau_{triangle} = 76.86$ MPa

6.23. Compare the shear stress and the unit angle of twist for three thin-wall sections: a circular tube, a square tube, and an equilateral triangle. The three sections have equal wall thicknesses and equal perimeters.

Ans. $\tau_{\text{square}} = 1.27\,\tau_{\text{circle}}$; $\tau_{\text{triangle}} = 1.65\,\tau_{\text{circle}}$; $\theta_{\text{square}} = 1.62\,\theta_{\text{circle}}$; $\theta_{\text{triangle}} = 2.74\,\theta_{\text{circle}}$

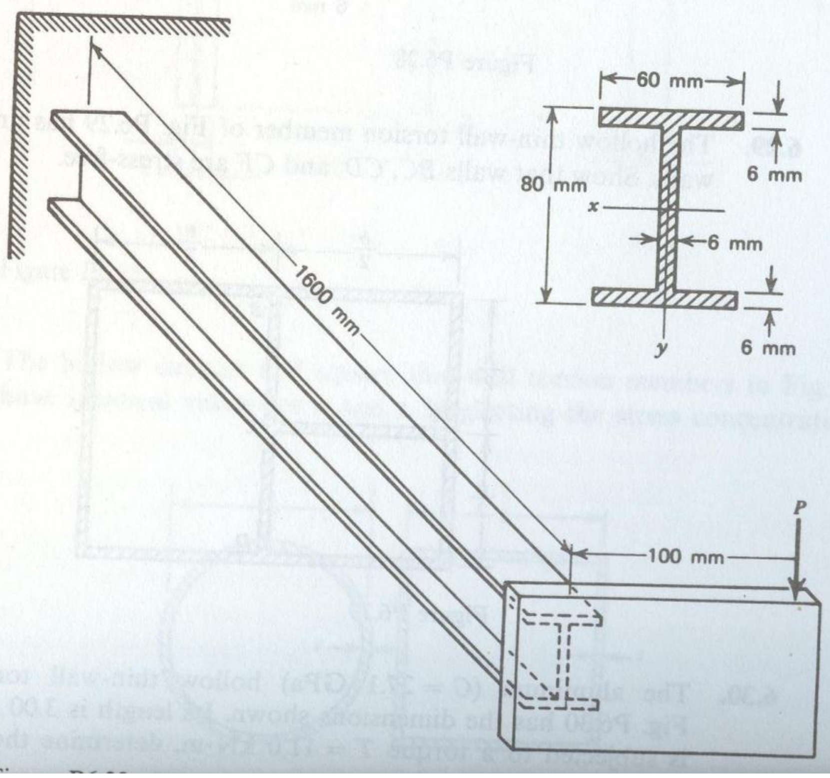
شماره۷:

6.7. A torsion member has an elliptical cross section with major and minor dimensions of 50.0 mm and 30.0 mm, respectively. The yield stress of the material in the torsion member is $Y = 400$ MPa. Determine the maximum torque that can be applied to the torsion member based on a factor of safety of $SF = 1.85$ using the maximum shear stress criterion of failure

... the I-beam.

6.32. The I-beam in Fig. P6.32 is an aluminum alloy ($E = 72.0$ GPa and $G =$ 27.1 GPa) extruded section. It is fixed at the wall and attached rigidly to the thick massive plate at the other end. Determine the magnitude of P for $\sigma_{\text{max}} = 160$ MPa.

Ans. $P = 1.095$ kN



شماره۸:

6.10. A rectangular bar has a cross section such that $b/h = k$, and it is subjected to a twisting moment T. A cylindrical bar of diameter d is also subjected to T. Show that the maximum shear stresses in the two bars are equal, provided $d = 3.441h(kk_2)^{1/3}$ and the bars remain elastic.

2. Show that $\phi = A(r^2 - a^2)$ solves the torsion problem for the solid or hollow circular shaft. Determine A in terms of $G\theta$. , evaluate the maximum shearing stress and the torsional rigidity in terms of M_t for the solid shaft, and verify that the results are in agreement with those given in any text